Electronic Supplementary Information
for
Challenges in Realizing a Self-Contained Hydraulically-Driven Contractile Fiber Actuator
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Detailed Methods

Calculations were performed in Excel to examine the strains that are achievable in each configuration, making the simplest possible estimates.

2MM Configuration

The initial uninflated volumes of the actuator V_{ai} and reservoir V_{ri} were taken as that of cylinders,

(1) $$V_{ai} = \pi R_{ai}^2 L_{ai} \; ; \; V_{ri} = \pi R_{ri}^2 L_{ri}$$

where the R_i are the initial radii and the L_i the initial lengths. This corresponds to both bladders being maximally filled without overpressure.

In long actuators, the McKibben muscle can be treated as maintaining a cylindrical shape, and the conical ends handled with an empirical fudge factor that corrects the strain [5]. However, in the configurations examined here, we were primarily concerned with strain, which requires short actuators, as shown below. Therefore, the inflated volume of the actuator was treated as a barrel whose volume was estimated as the weighted average of two cylinders of radius R_{ai} and $R_{ai}+\Delta R_a$:

(2) $$V_a = V_{ai} + \Delta V = \pi (2L_a) \frac{R_{ai}^2 + 2(R_{ai} + \Delta R_a)^2}{3}$$

For a given lengthwise perimeter P, the actuator volume reaches a maximum, limiting the strain.

Likewise, the deflated volume of the reservoir bladder was set by subtracting the volume of fluid entering the actuator, with the shape being treated as an inward-bending barrel:

(3) $$V_r = V_{ri} - \Delta V = \pi (2L_r) \frac{R_{ri}^2 + 2(R_{ri} - \Delta R_r)^2}{3}$$

The volume is a minimum when the walls of the bladder touch in the center, $R_{ri} = \Delta R_i$. This will also limit the strain.

The general approach taken in these calculations was to systematically reduce the actuator length L_a and at each step calculate the change in actuator radius ΔR_a based on the assumption of a constant lengthwise actuator perimeter P (given a non-stretchable sleeve [5]), then calculate the volume of the

actuator to find ΔV. From the change in reservoir volume, the change in its radius was next determined, which was straightforward because its length was constant.

In its initial state, with no bulging, the length *P* of the actuator sleeve (Figure 2 of the main text) is

(4) $P = 2L_{ai}$,

where L_{ai} is half the actuator length. For this calculation, the shape of the bulge is approximated as that of an ellipse. The method of obtaining ΔR_o as the tube begins to bulge is less straightforward than one would like. The perimeter of an ellipse must be approximated, but the most accurate expression contains cross-terms and is not amenable to determining an unknown radius from a known perimeter. The simplest estimate for the perimeter is

(5) $$P \approx 2\pi\sqrt{\frac{a^2+b^2}{2}},$$

which would give for the bulge radius ΔR_o, using half the ellipse perimeter,

(6) $$\Delta R_a = \sqrt{\frac{2P^2}{\pi^2} - L_a^2}\,.$$

However, this expression is inaccurate for flattened ellipses – the case at the beginning when the tubular actuator begins to bulge – and does not yield any value when the ratio of *a*:*b* exceeds 9:1 since the square root is negative. Therefore, at low inflation the radius was assumed to increase as the square root of ΔL times a factor *f*, where *f* was chosen case-by-case for a smooth transition between the two expressions. The calculated radius as a function of actuator length is shown in Figure 1 for $\Delta R_a = f\sqrt{\Delta L}$, the estimate of equation (6), and a curve that switches from the former to the latter at *L* = 0.75, when equation (6) gives a reasonable estimate.

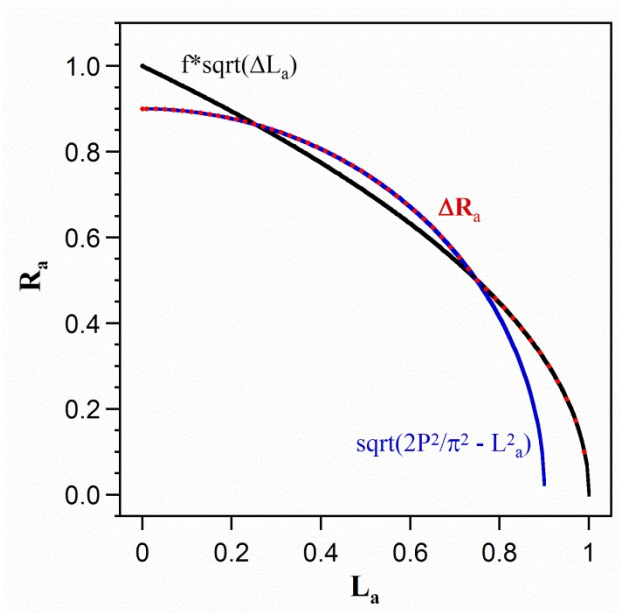

Figure 1. Estimated radius of ellipse based on equation (6) (blue), $R = \sqrt{\Delta L}$ (black), and a switch from the latter to the former (red).

The system was subject to several constraints. The length of actuator must stay above zero; the volume of actuator must continue to increase as its length contracts; the reservoir volume cannot go below zero; and the reservoir walls cannot touch (pinching off the fluid flow).

The connector and pump lengths were neglected, although this appreciably overestimates the strain. The assumption of a constant length *P* also leads to inaccuracies in the predicted strain, since typical sleeve weaves have fibers that cross at an angle. Given the assumptions and estimates, the results presented here are qualitative and best-case.

MM-CT Configuration

The actuator piece of the calculations for the MM-CT were handled the same way as above. The concertinaed tube was treated as a series of *n* identical segments having the shapes of truncated cones, for which the volume is:

(7) $$V_{trunc.cone} = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right),$$

where h is the truncated cone height and the r_i are the radius at the top and bottom. Therefore, the volume of the reservoir is:

(8) $$V_r = nV_{trunc.cone} = \frac{\pi n L_n}{3}\left(R_{ri}^2 + R_{ri} r + r^2\right)$$

where L_n is segment length and r is the smaller radius. The L_n of the two outermost segments were the same as for the other segments, so the larger radius was always R_{ri}. This geometry requires an even number of segments. The concertinaed walls make an angle θ from their original horizontal position when they fold. Half the folded reservoir length is thus:

(9) $L_r = L_{ri}\cos\theta = (nL_n/2)\cos\theta.$

The segment length L_n must be less than R_{ri} in order to avoid the two walls touching when they fold; it was set to $0.9R_{ri}$. The small radius was then

(10) $r = R_{ri} - L_n\sin\theta = R_{ri}(1 - 0.9\sin\theta).$

The maximum bending angle was taken as 75°. Given a maximum ΔV for the actuator, the optimal value for the initial reservoir volume is then just slightly larger than ΔV. To find the maximum strain, the difference in extended and folded volumes was set to equal the change in volume of the actuator for various combinations of initial reservoir lengths and radii, choosing values to give *n* even.

Further Results

2MM Configuration

Strain in the 2MM system is shown as a function of decreasing L_a (corresponding to increasing V_a) in Figure 2 for two different initial reservoir lengths L_{ri}.

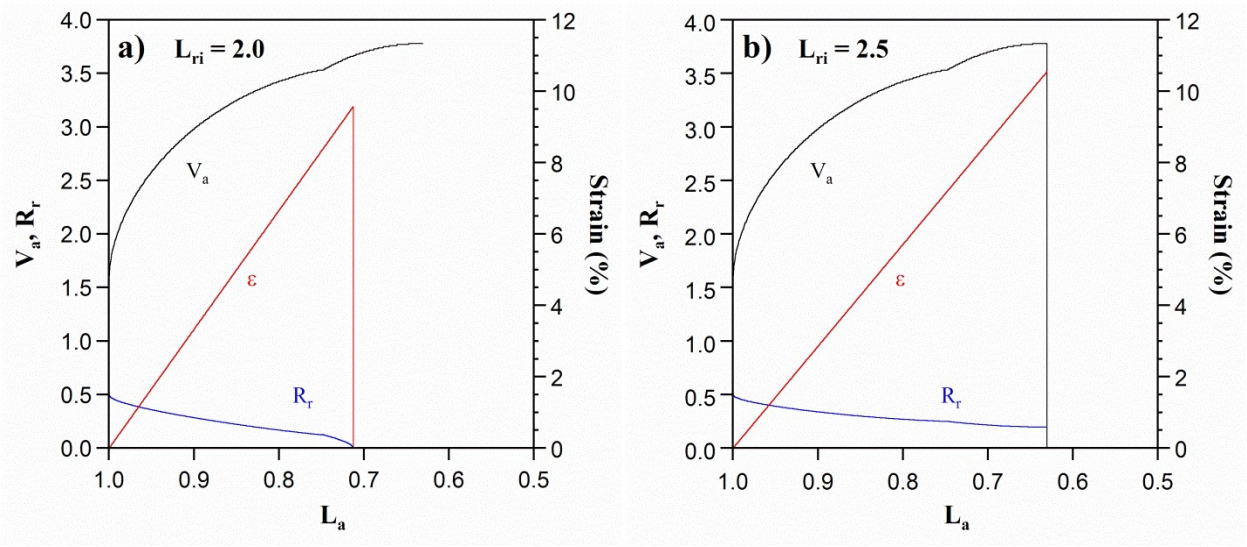

Figure 2. Example curves for the baseline case illustrated in Figure 2 of the main text. Note the reversal of the *x*-axis because increasing actuator volume V_a (black curve) corresponds to decreasing L_a. a) The separation of the reservoir walls, R_{ri} - ΔR_i (blue), decreases with decreasing actuator length. Fluid is taken to stop pumping when this pinched-off condition occurs, even though there is still some fluid in the reservoir, limiting the strain (red). b) For larger reservoirs, strain is limited by the actuator becoming maximally filled with fluid (red). (The discontinuity in the curvature of V_a is due to the correction of Figure 1.)

Strain in the 2MM system as a function of reservoir length L_r for one reservoir versus two reservoirs of half the length.

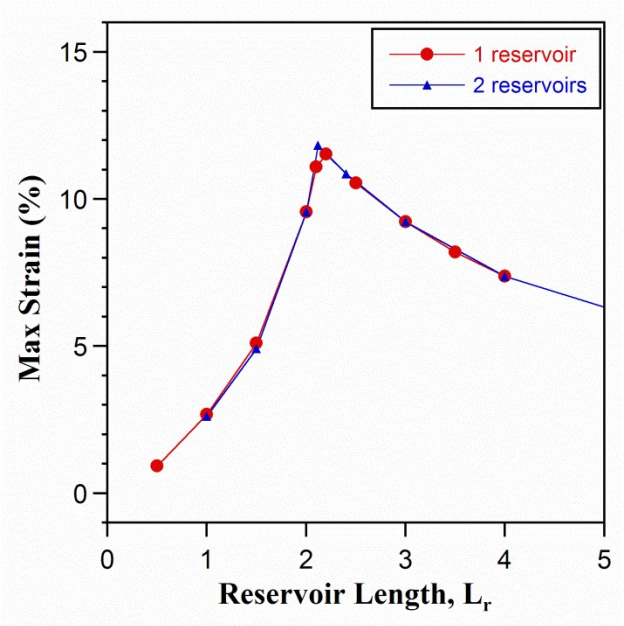

Figure 3. Maximum strain as a function of total reservoir length for one versus two reservoirs.